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Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2017
First Degree Programme under CBCSS
MATHEMATICS
Core Course – XI
MM 1643 : Complex Analysis – II
(2014 Admn.)

Time: 3 Hours

Max. Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry 1 mark **each**.

1. Write the power series representation of $f(z) = \frac{1}{1+z}$.

2. Find b_k, if f(z) is analytic in D(a, r) and $f(z) = \sum_{k=0}^{\infty} b_k (z-a)^k$.

3. Write the singularities of the function $\frac{z}{\cos z}$.

4. Define removable singularity.

5. What is the nature of singularity of the function $\frac{\sinh z}{z^4}$?

6. Define a pole of order m for f(z).

7. Find the residue of $\frac{e^2}{z^2+\pi^2}$ at $z = \pi i$.

8. Find the principal value of $\int_{-\infty}^{\infty} x\,dx$.

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9. What is the order of the zero of the function $f(z) = z(e^z - 1)$?

10. What is the type of singularity of the function $\sin \frac{1}{1-z}$ at $z = 1$?

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. They carry **2** mark

11. Expand $f(z) = \frac{1}{z^2}$ as a power series in the disc $|z + 1| < 1$.

12. Evaluate $\int_C \frac{5z-2}{z(z-1)} dz$ where C is the circle $|z| = 2$.

13. Find the nature and location of the singularities of $\frac{z^2-1}{(z-1)^3}$.

14. Find the residue of $z \cos \frac{1}{z}$.

15. Determine the residue at $z = 0$ of $\frac{1-\cosh z}{z^3}$.

16. Write the principal part of the function $\exp\left(\frac{1}{z}\right)$.

17. State Cauchy's Integral formula.

18. State Jordan's Lemma.

19. Find the residue of $f(z) = \frac{\cot z}{z^4}$ at $z = 0$.

20. Evaluate $\int_C \frac{\exp(-z)}{z^2} dz$ where C is the circle $|z| = 3$.

21. Find $\sum_{n=1}^{\infty} \frac{1}{n^2}$

22. Find a power series representation for $\frac{1}{z^2}$ near $z = 3$.

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SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. They carry **4** marks **each**.

23. State and prove Cauchy's Residue Theorem.

24. Evaluate $\int_C \frac{z+1}{z^2-2z}\,dz$, where C is $|z| = 3$ using Cauchy's Residue Theorem.

25. Using a single residue, evaluate the integral $\int_C \frac{1}{1+z^2}\,dz$ where C is $|z| = 2$.

26. Using Cauchy's integral formula evaluate $\int_C \frac{z}{(9-z^2)(1+z)}\,dz$ where C is $|z| = 2$.

27. Describe three types of isolated singular points with examples.

28. Prove that an isolated singular point z_0 of a function f is a pole of order m if and only if f(z) can be written in the form $\frac{\phi(z)}{(z-z_0)^m}$ where $\phi(z)$ is analytic and non zero at z_0.

29. Evaluate $\sum_{n=0}^{\infty} {}^{2n}C_n \left(\frac{1}{5^n}\right)$.

30. Suppose that two functions p and q are analytic at z_0 and $p(z_0) \neq 0$ and q has a zero of order m at z_0. Prove that $\frac{p(z)}{q(z)}$ has a pole of order m at z_0.

31. Evaluate $\int_0^{2\pi} \frac{1}{5+4\cos x}\,dx$.

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SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. They carry **15** marks **each**.

32. a) Use Cauchy's Residue theorem to evaluate $\int_C \frac{4z^3 + 2z}{z^4 + z^2 + 1} dz$, where C is $|z| = 2$

b) Find $\text{Res}_{z=n\pi} \cot z$.

33. a) Evaluate $\int_0^\infty \frac{1}{x^2 + 1} dx$.

b) Evaluate $\int_0^\infty \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx$.

34. a) Evaluate $\int_0^\infty \frac{\cos 2x}{x^2 + 1} dx$.

b) Evaluate $\int_{-\infty}^\infty \frac{x \sin 2x}{x^2 + 4} dx$.

35. a) Evaluate the Cauchy Principal values of the improper integral

$$\int_{-\infty}^\infty \frac{\sin x}{x^2 + 4x + 5} dx.$$

b) Evaluate $\int_{-\infty}^\infty \frac{x \sin x}{x^2 + 2x + 2} dx$.

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